MSRI SUMMER SCHOOL SYLLABUS

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Outline: We will be running three parallel courses: Riemann Surfaces (beginning graduate level), Geometric Analysis, and Complex Geometry (both intermediate/advanced graduate). Each course will consist of one lecture and one Q&A/problem session per day for the whole two weeks.

1. Riemann Surfaces (Bernstein)

Prerequisites: Knowledge of basic complex analysis—at the level of Ahlfors, *Complex Analysis*, Chapters 1-5—will be assumed. Some basic familiarity with (abstract) surface theory and differential forms will be helpful. However, I will review this material as needed.

Reading: The main text will be
- Donaldson, *Riemann Surfaces*; get at `http://www2.imperial.ac.uk/~skdona/RSPREF.PDF`.

Other useful references:
- Farkas and Kra, *Riemann Surfaces*; a classical text on the subject.
- Miranda, *Algebraic Curves and Riemann Surfaces*; a more algebraic perspective.

Week 1: Introduction to Riemann Surfaces
Surfaces and Topology
Riemann Surfaces and Holomorphic Maps
Maps between Riemann Surfaces
Calculus on Riemann Surfaces
De Rham Cohomology

Week 2: Geometric Analysis on Riemann Surfaces
Elliptic Functions and Integrals
Meromorphic Functions
Inverting the Laplacian
The Uniformization Theorem
Riemann Surfaces and Minimal Surfaces

2. Geometric Analysis (Naber)

Prerequisites: Basics of manifolds, tensors, and differential forms. Basics of pde theory, for instance Evans's book *Partial Differential Equations*, in particular those chapters on second order elliptic and parabolic equations. Familiarity with exponential maps, injectivity radius, and geodesics would be helpful, for instance chapter one of Jost's book *Riemannian Geometry and Geometric Analysis* is more than sufficient.

Date: May 8, 2014.

Reading: The main source will be Petersen's book on Riemannian Geometry. We will also rely on Jost's *Riemannian Geometry and Geometric Analysis*, and on the book by Cheeger *Degeneration of Riemannian Metrics Under Ricci Curvature Bounds*. More advanced topics will use relevant papers in the field.

Week 1: Introduction to Geometric Analysis
Review of Manifolds and Smooth Structure
Introduction to Curvature and Geodesic Coordinates
Laplacians and Harmonic Coordinates
Heat Kernels and Geometry
Sectional Curvature and Finite Diffeomorphism Theorems

Week 2: Topics in Regularity Theory
Ricci Curvature, Volume Monotonicity and Rigidity Theorems
Ricci Curvature and Almost Rigidity Theorems
Lower Ricci Curvature and Stratification Theorems
Bounded Ricci Curvature and ε-regularity Theorems
Outline of Regularity Theory for Einstein Manifolds

3. Complex Geometry (Hein)

Prerequisites: - Basics of manifolds, tensor fields, differential forms, etc. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, Chapters 1, 2, 4, 6, contains all we need and much more.
- Basic complex analysis as in Stein & Shakarchi, *Complex Analysis*, Chapters 1, 2, 3, 8.

Reading: - Huybrechts, *Complex Geometry*, is an excellent basic textbook with exercises.
- Lecture notes by Joel Fine: http://homepages.ulb.ac.be/~joelfine/papers.html#survey.
- Complex Monge-Ampère: http://gamma.im.uj.edu.pl/~blocki/publ/ln/tln.pdf.
- For the end of Week 2: http://arxiv.org/pdf/0803.0985.pdf, Section 5.

Week 1: Introduction to Complex Geometry
Holomorphic Functions and Complex Calculus
Complex Manifolds
Holomorphic Line Bundles
Pseudoconvexity and Pseudoconcavity
The Kodaira Embedding Theorem

Week 2: Topics in Kähler-Einstein Manifolds
Kähler Manifolds
Ricci Curvature and the Complex Monge-Ampère Equation
Examples of Ricci-flat Spaces
Basic Estimates for the Complex Monge-Ampère Equation
The Mukai-Umemura Manifold