

## Exercise Sheet 7

1. Prove a non-trivial upper bound for

$$\sum_{n \leq x} \Lambda(n) e\left(\frac{1 + \sqrt{5}}{2} n\right).$$

2. Show that

$$\int_0^T \left| \sum_{n=1}^N a(n) n^{it} \right|^2 dt \leq \sum_{n=1}^N |a(n)|^2 (T + O(n)).$$

3. Define  $p(n)$  to be the number of partitions of  $n$ . Use a contour integral to estimate the value of  $p(n)$ .

4. Define

$$\mathcal{P}_{x,y} := \left\{ p \leq \sqrt{x} \text{ such that } \forall n \leq y, \left(\frac{n}{p}\right) = 1 \right\}$$

- (a) Find an upper bound for  $\psi(x; y)$  in terms of  $\mathcal{P}_{x,y}$  using the large sieve.  
(b) Show that for all  $\varepsilon > 0$ ,

$$|\mathcal{P}_{x,x^\varepsilon}| \ll_\varepsilon 1.$$