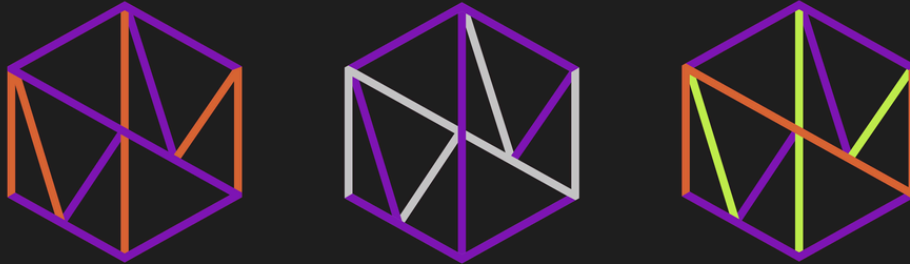


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# ADJOINT 2023



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## Notice of Institute Name Change

Beginning in 2022-2023, the Mathematical Sciences Research Institute (MSRI) has become the Simons Laufer Mathematical Sciences Institute (SLMath). Researchers refer to the institute as MSRI at times in this report.

[www.slmath.org](http://www.slmath.org)



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## **ADJOINT 2023**

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### **A. Introduction**

The **ADJOINT Mathematics Workshop** is a yearlong program that provides opportunities for U.S. mathematicians – especially those from the African Diaspora – to form collaborations with distinguished African American research leaders on topics at the forefront of mathematical and statistical research.

Beginning with an intensive two-week summer session at SLMath, researchers work in small groups under the guidance of some of the nation's foremost mathematicians and statisticians to expand their research portfolios into new areas. Throughout the following academic year, the program provides conference and travel support to increase opportunities for collaboration, maximize researcher visibility, and engender a sense of community among researchers.

### **B. Structure and Researchers:**

Four research groups comprising 12 researchers (including the four Research Leaders) participated in ADJOINT 2023. The cohort was broadly diverse in terms of gender, career stage, geographic location, and national origin. The researchers hold positions ranging from postdoctoral fellowships to full professorships; hailing from liberal-arts to research-intensive institutions as well as from minority-serving to predominantly white institutions. The research projects spanned algebraic geometry, partial differential equations, mathematical biology, and biostatistics.

The two-week intensive summer session took place June 19 to June 30, 2023. All researchers were able to participate in person.

As the first day of ADJOINT fell on the Juneteenth holiday, SLMath reserved a hotel conference room for the research teams. The program opened with a welcome session involving all researchers and onsite members of the ADJOINT Directorate. All attendees introduced themselves and reviewed the schedule for the two-week intensive workshop. One element of the welcome session was a Juneteenth celebration, facilitated by ADJOINT directors Anisah Nu'Man and Caleb Ashley. Beginning with a brief presentation on the significance of Juneteenth (a commemoration of the end of slavery in the U.S. after the Civil War -- June 19, 1865), the event concluded with a Pan-African libation ceremony wherein a California redwood bonsai tree was dedicated to longevity of ADJOINT and the enduring contributions of members of the African Diaspora to the mathematical sciences.

On the second day of the program, SLMath Librarian Linda Riewe gave a library orientation. There was also a professional development workshop entitled *Developing a Strategic Plan for One's Career* in which the four research leaders, Folashade Agosto, Jonathan Esole, Donald Martin, and Nsoki Mavinga, shared their mathematical journeys and tips for developing a plan and timeline for one's career. Groups spent the remainder of the first week advancing their projects.

The recommended format for ADJOINT was that each day begin with a period of "training," crosstalk, updates, debrief, feedback, strategy adjustment, etc.; however, Research Leaders had autonomy over their daily collaborative sessions. Lunches and coffee breaks were unstructured and held at overlapping times with other programs at SLMath to allow for interaction between participants of the different summer activities, including the researchers from Summer Research (for Women) in Mathematics (SRiM). Additionally, a joint networking dinner, catered by Jamaican-American chef Nigel Jones of Kingston 11 Cuisine was hosted on the outdoor terrace at SLMath. At this dinner, all researchers and several of their family members engaged in extended informal conversations and deepened their sense of camaraderie. This social event was captured by a professional photographer who also took headshots of the ADJOINT researchers.

During the second week of the ADJOINT 2024 two-week intensive period, researchers participated in two additional professional development workshops. One workshop entitled Grant Writing and Funding Opportunities included a virtual presentation from Alfred Hall with the National Science Foundation and another a second workshop entitled Developing an Online Research Community focusing creating a plan to continue their research projects remotely. Another activity held during the second week of ADJOINT was a joint panel with MSRI-UP on mentorship. The panelists for this session included: Sofia Martinez-Alberga (graduate student), Ana Wright (postdoc), Dwight A. Williams (ADJOINT 2024 researcher), and Nsoki Mavinga (ADJOINT research leader). Common activities between ADJOINT researchers and the MSRI-UP undergraduate student participants provided opportunities for networking and fellowship between the two groups.

Two research sharing sessions took place: one at the end of the first week, and one at the end of the second week.

### **C. ADJOINT 2023 Research Groups**

Listed below are the three research groups that comprised the ADJOINT 2023 program, including a summary of their research projects.

#### **“Ticks, Fire and Control: Implementing prescribed fire and control measures at ticks invasion front”**

Research Leader: Dr. Folashade Agosto (University of Kansas)

Research Group: Kwadwo Antwi-Fordjour (institution) and Isabelle Kemajou-Brown (Morgan State University)

**Description:** In recent times, ticks have been expanding their habitat ranges due to climate change, among other factors, leading to increased tick-borne illness risks in the United States. Thus, it is important to find a practical and cost-efficient way of controlling and managing ticks. Prescribed burns are intentional fires used for land and forest management; they are time and

cost efficient, since they can be applied across large amounts of land. They can also be effective in controlling tick populations.

### **“Elliptic fibrations, flops, and collisions of singularities”**

Research Leader: Dr. Jonathan Esole (Northeastern University)

Research Group: Saber Amhed (Hamilton College) and Dwight Anderson Williams (Pomona College)

**Description:** Elliptic fibrations are beautiful and elegant geometries at the interface of algebraic geometry, number theory, and string theory. While ubiquitous, they still have many secrets to teach us. In particular, their uses by physicists indicate deep connections of their birational geometry with representation theory and hyperplane arrangements of weight systems. The geometric singular fibers over generic points of the discriminant locus of a smooth elliptic fibration are famously classified by Kodaira and Neron. They have dual graphs corresponding to twisted affine Dynkin diagrams. Under mild conditions, an elliptic fibration is birationally equivalent to a (singular) Weierstrass model. Two different crepant resolutions are connected by a sequence of flops and have some matching topological invariants, such as their Euler characteristics. We are interested in the structure of crepant resolutions of Weierstrass models of elliptic fibrations corresponding to given types of Kodaira fibers. We will explore their flops and compute their topological invariants.

### **"Inference and applications of sparse Markov model"**

Research Leader: Dr. Donald Eugen Kemp Martin (North Carolina State University)

Research Group: Eli Alhajjar (United States Military Academy) and Nonhle Channon Mdxiniso (Rochester Institute of Technology)

**Description:** Analysis of a categorical time series is facilitated by a model that captures the statistical properties of the sequence, while being simple enough so that statistical analysis is feasible. Markov models of an order of dependence  $m$  serve this purpose. However, the number of parameters in a Markov model is exponential in  $m$ , leading to variance considerations because many parameters need to be estimated from data. Thus first-order Markov chains ( $m = 1$ ) are more frequently used. Yet using a lower-order model when higher-order dependence is called for leads to bias. Sparse Markov models help with this bias-variance trade-off. A sparse Markov model (SMM) is a higher-order Markov model for which conditioning  $m$ -tuple histories are grouped into classes such that the conditional probability distribution is constant over  $m$ -tuples in the same class. The clustering reduces the number of parameters.

### **"Nonlinear elliptic partial differential equations with nonlinear boundary conditions"**

Research Leader: Dr. Nsoki Mavinga (Swarthmore College)

Research Group: Timothy Ira Myers (Howard University) and M.N. Nkashama (University of Alabama at Birmingham)

**Description:** Since Bernstein's pioneering work in nonlinear elliptic equations in 1906 and Leray's work on hydrodynamical problems in 1933, there has been a significant amount of interest in the

study of nonlinear elliptic partial differential equations. Henceforth, there has been a tremendous emphasis in studying partial differential equations that enable understanding and modeling of nonlinear processes such as chemical, biological or ecological processes. This area has become central in modern-day mathematical research. The study of nonlinear elliptic partial differential equations with linear boundary conditions is relatively well studied and the literature is extensive. However, problems with nonlinear boundary conditions have recently gained attention due to their importance in the study of several branches of pure and applied mathematics such as the theory of quasiregular and quasiconformal mappings in Riemannian manifolds with boundary, non-Newtonian fluids, nonlinear elasticity and glaciology, among others. In this research project, we will focus on the qualitative (analytical) study of nonlinear elliptic equations subject to nonlinear boundary conditions. In particular, we consider problems with nonlinear reactions in the interior as well as on the boundary of the domain in consideration. We will investigate the existence, nonexistence, multiplicity, and properties of solutions. In the treatment of the proposed problems, new analytical tools will be developed as well as utilize the methods of nonlinear functional analysis such as sub-supersolution method, degree theory, bifurcation theory and fixed points theorems.

#### **D. Highlights from Research Leader Reports**

Research Leaders submitted brief reports within one month after the two-week intensive period. The reports summarized (actual and planned) activities during the two-week research session and beyond.

All groups had intensive “training” sessions to get everyone to the same level and ready to obtain/derive new results, although the format differed slightly between groups. Some groups shared lecture-style materials from representative books and others typed up their meeting notes using LaTeX. Beyond the two-week intensive period, groups have met with varying frequency from weekly to every two weeks. All groups have begun work on manuscripts and/or presentations and have developed plans to continue their projects. Each group’s plans have been summarized below:

**Dr. Augusto** and her team remain actively engaged in the project initiated during ADJOINT 2023. Currently they are exploring the probability of disease extinction and persistence using transition probability and Markov property from the stochastic system developed during the two weeks in residence at SLMath. ADJOINT researcher Dr. Isabella Kemajou-Brown’s experience with such systems allowed her to lead the team’s initial efforts in that direction. Additionally, the team addressed their original research question involving location of additional tick control measures and prescribed fire; this system involves a PDE and impulsive systems. A manuscript authored by Kwadwo Antwi-Fordjour, Isabella Kemajou-Brown, and Folashade B. Augusto, titled “Modeling the effect of lethal and non-lethal predation on the dynamics of tick-borne disease,” has been submitted to *Mathematical Biosciences*. Another manuscript authored by the above researchers together with Dr. Nsoki Mavinga focusing on modeling the *stochastic* effect of lethal and non-lethal predation on disease dynamics is currently in progress. The collaboration with Dr. Mavinga, another 2023 research leader, demonstrates the professional networking and cross-pollination of expertise that takes place during ADJOINT. Dr. Antwi-Fordjour delivered a talk on “Dual Fear Phenomenon in an eco-epidemiological model with prey aggregation” at the SLMath Special Session on Summer Research in Mathematics (SRiM — another flagship SLMath program) at the JMM 2024.

**Dr. Esole's** group has made significant progress studying the network of crepant resolutions of elliptic fibrations defined by a Weierstrass model with a generic fiber of Kodaira type IV\*-split over the generic point of a smooth divisor in the base of the elliptic fibration. A conjecture in string theory conjectures that there are twelve such crepant resolutions connected by flops such that the network of flops of a hyperplane arrangement defined by the hyperplanes defined by the kernel of the weights of the minuscule representation of E6 restricted in the dual fundamental Weyl chamber. The group constructs half of the crepant resolutions and carefully studies their geometries, and has proven that they match what is expected from the representation theory data. Saber Ahmed presented these results at the Joint Mathematics Meeting in January 2024. Drs. Esole and Ahmed have kept in close collaboration since the 2 week-summer program, meeting weekly during the summer and naturally somewhat less frequently during the academic year. They are currently finalizing the draft of the resultant paper. The next step will be to explore the crepant resolutions of the E7 case, for which only half of the crepant resolutions are known and share many complications of the E6 case. The E7 and E6 cases are the only ones left to understand fully.

While at SLMath, **Dr. Martin** and his team worked on a method of improving efficiency of techniques for computing the distribution of a pattern statistic. The underlying sequence is assumed to be Markovian. Whereas a Markov chain approach to computation yields an efficient computation mechanism to compute a single distribution, if distributions are needed for many values of input probabilities, the entire computation needs to be repeated many times. The method forwarded in this work avoids the need to redo recursive updates of probabilities. Instead, one sequentially updates counts of data strings with various values of sufficient statistics through a single pass through the sequence; the final counts are then used to reconstruct probabilities for the many input parameters, improving efficiency. Dr. Martin will present the general method at the Joint Statistical Meetings in Portland, OR (in a talk entitled "Efficient computation of pattern statistics for many different input probabilities"), while Dr. Mdziniso will present an application in the talk entitled "Efficient inference for start-up demonstration tests." Dr. Alhajjar is a co-author of the overall paper from which both talks emanate.

Following the retirement of Dr. Jacqueline Hughes-Oliver in 2023, Dr. Martin was elected to the ADJOINT Directorate in 2024, a testament to the success of the program's networking structure. He will contribute significant expertise in the statistical sciences and deepen the program's reach into the statistical community.

**Dr. Mavinga** and her team continue to advance their research agenda. All members of the group have presented their findings at conferences and events. Dr. Myers delivered talks on "A Constructive Existence Result for a Nonlinear Elliptic PDE with Nonlinear Boundary Condition: Monotone Case" at the 41st Southeastern Atlantic Conference on Differential Equations, the Joint Mathematics Meetings (JMM) 2024, and a Colloquium Series at Howard University. Dr. Nkashama presented on "Nonlinear Elliptic Equations with Growth involving Critical Sobolev Exponents" at the JMM as well. Dr. Mavinga delivered talks on "Nonlinear Elliptic Problems with Growth up to Critical Exponents" at several conferences, including the 2023 Annual Meeting of SIAM-NNP, the International Conference on Differential Equations, and a CADEDIF-Workshop on Differential Equations at Universidad Complutense de Madrid. She has also been invited to speak at the September 2024 AMS Sectional Meeting special session on Harmonic Analysis, Geometric Measure Theory, and PDEs. The group's fruitful collaboration has led to the preparation of a forthcoming paper entitled "Weak Solutions of Nonlinear Elliptic Problems with Growth up to Critical Sobolev Exponents." They have also participated in networking events and

professional development opportunities. Drs. Mavinga and Nkashama met during a research visit to the University of Alabama at Birmingham, October 2-9, 2023. Dr. Mavinga also attended the Mid-Atlantic Mathematical Biology (MAMBio) Community Workshop and is scheduled to participate in the MAA's OPEN Math Summer 2024 Workshop in July 2024. In terms of grants, Dr. Mavinga submitted a proposal for the AMS-Simons Research Enhancement Grants for PUI in March 2024. As of May 2024, the group continues to meet regularly via Zoom.

### **E. Continued Collaboration, Research, and Career Development Activities:**

For the fourth year in a row, ADJOINT directors have organized a special session at the Joint Mathematics Meeting (JMM) to showcase the research of ADJOINT researchers. The annual JMM attracts thousands of mathematicians each year. For the 2024 JMM special session, a total of 8 researchers from past ADJOINT cohorts gave talks:

- ADJOINT 2023: Nsoki Mavinga (Swarthmore College), Saber Ahmed (Hamilton College), Kwadwo Antwi-Fordjour (Samford University), Timothy Myers (Howard University),
- ADJOINT 2021: Emille Davies Lawrence (University of San Francisco),
- ADJOINT 2020: Eddy Kwessi (Trinity University), Henok Mawi (Howard University),
- ADJOINT 2020: Lawrence Udeigwe (Manhattan College)

Scheduled talks titles and abstracts can be found here:

[https://jointmathematicsmeetings.org/meetings/national/jmm2024/2300\\_program\\_slmss1.html#title](https://jointmathematicsmeetings.org/meetings/national/jmm2024/2300_program_slmss1.html#title)

### **F. Program Evaluation:**

At the close of the two-week intensive period, all researchers were given a week to complete an online retrospective pre/post survey to provide feedback on their experiences. All researchers and research leaders provided responses. The surveys were administered through SLMath.

For a second-year researchers were asked to select among 12 choices why they joined ADJOINT. Among responses, *all* researchers indicated the following four reasons for joining ADJOINT:

- To begin a new group project that will move towards publication.
- To increase my long-term research output (papers, presentations, etc.).
- To collaborate with people who I would not otherwise have access to.
- To initiate or strengthen connections within the African American mathematical and statistical sciences community.

When asked about the extent to which ADJOINT met their expectations in the 12 choices (measured using a three-point response scale: 'Fell short of,' 'Met,' or 'Exceeded' expectations) program researchers' expectations were met (or exceeded) 11 out of the 12 categories, including the four top choices for joining ADJOINT listed above. The four Research Leaders were asked the same question and indicated their top two reasons for joining ADJOINT were:

- To begin a new group project that will move towards publication.
- To initiate or strengthen connections within the African American mathematical and statistical sciences community.

Research Leaders similarly expressed that their expectations were met (or exceeded) in 11 out of the 12 categories, including their top two reasons for joining ADJOINT.

The only area (1 out of 12) where the two-week intensive session fell slightly short of expectations was: "To learn about grant funding opportunities for research." ADJOINT 2024 will emphasize funding opportunities, including the new National Science Foundation (NSF) Partnerships for Research Innovation in the Mathematical Sciences (PRIMES) grant, which "increases diversity through building and growing research capacity and output in the mathematical sciences at minority serving institutions (MSI) by supporting members of a math/stat department at an MSI through year-long research leaves, during which time researchers visit one of the [NSF]-supported research institutes (for a thematic program, for workshops of any length, or for a long-term stay)."

Some other findings from the 2023 Retrospective Pre/Post Survey analysis are summarized below.

- Researchers and Research Leaders are quite likely to recommend ADJOINT to others, with an average rating of 4.82 (out of 5).
- Researchers and Research Leaders report being highly likely to continue collaborating with members of their research group, with an average rating of 4.91.
- Researchers and Research Leaders all reported that SLMath should continue the ADJOINT program.
- Researchers were very satisfied with their Research Leader and group members, reporting average rating of a perfect 5.0 for helpfulness of Research Leader, 4.5 for effectiveness of Research Leader, 4.7 for satisfaction with contributions from other group members, and 4.7 for enthusiasm to continue working with the group.
- Research Leaders were very positive about their group members, reporting average ratings of 4.4 on group member engagement, 4 for satisfaction with group member contributions, a perfect 5 for enthusiasm to continue working with the group, and confidence in plans for continuing collaboration, and a 4.6 for productive interaction among group members.
- Research Leaders believe the most likely research outputs would be a publication (rating of 5.0) or a conference proposal (5.0), and the least likely a grant proposal (3.8).
- Researchers and Research Leaders had mixed responses on the workshops offered. Workshop 1: "Mathematical Journey and Developing a Strategic Plan for one's Career" Works (rating 3.5), Workshop 2: "Grant Writing and Funding Opportunities" (rating 2.7), and Workshop 3: "Tips for Continuing Your ADJOINT Project Remotely" (rating 4.0). Researchers and Research Leaders would have liked to see a range of funding opportunities available since they all come from varying institutions, and not just HBCUs.
- The Research Shares received mixed reviews, with researchers finding more value than Research Leaders regarding clarifying opportunities for improvement within a group (3.7 versus 2.5), and the reverse for sparking ideas to advance one's own research interests (3.1 versus 3.5).
- When it comes to continuing collaborations with some or all of the members of their research groups, researchers and research leaders responded with an average of 4.91. As for initiating collaborations with any ADJOINT researcher (potentially across research groups) the average response was a 3.73.

- Comparing the time period BEFORE the two-week intensive ADJOINT workshop and NOW after the two-week workshop, researchers rated the following questions (measured on a five-point response scale: 1 being 'Not Strong' and 5 being 'Very Strong'):
  - How would you rate your connection to mathematical and statistical research scientists of the African Diaspora? (average 2.3 before, average 4.1 after)
  - How would you rate your social networks within the mathematical and statistical sciences communities? (average 2.6 before, average 3.7 after)
  - How would you rate your ability to bridge into a new area of mathematics or statistics? (average 3.1 before, average 4.2 after)
  - How would you rate your sense of being able to act independently to make choices that will influence your career? (average 3.5 before, average 4.2 after)
  - How would you rate your ability to advocate for your professional needs? (average 3 before, average 3.9 after)
  - How would you rate your ability to be more intentional with your time and engagements (e.g., avoiding overcommitting, engaging in activities that further career goals, scheduling time for research, etc.)? (average 3.1 before, average 4.2 after)

Baseline information was also collected on scholarly activities in several categories (e.g., attendance/presentation at conferences, journal articles, grant proposals, etc.), and this information will be crucial in tracking short- and long-term benefits of the ADJOINT program. Researchers also provided ideas for future professional development activities, and future Research Leaders and topic areas.